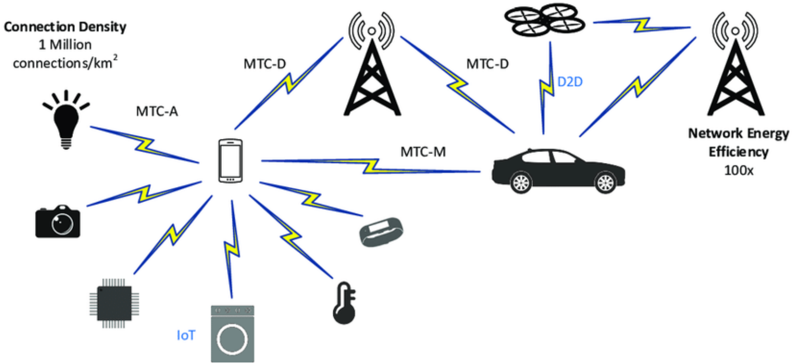
Coding in some 5G use cases

Tefjol Pllaha
Joint with R. Calderbank, O. Tirkkonen, R. Vehkalahti

Department of Mathematics
University of Nebraska - Lincoln

Massive Machine Type Communications (mMTC)



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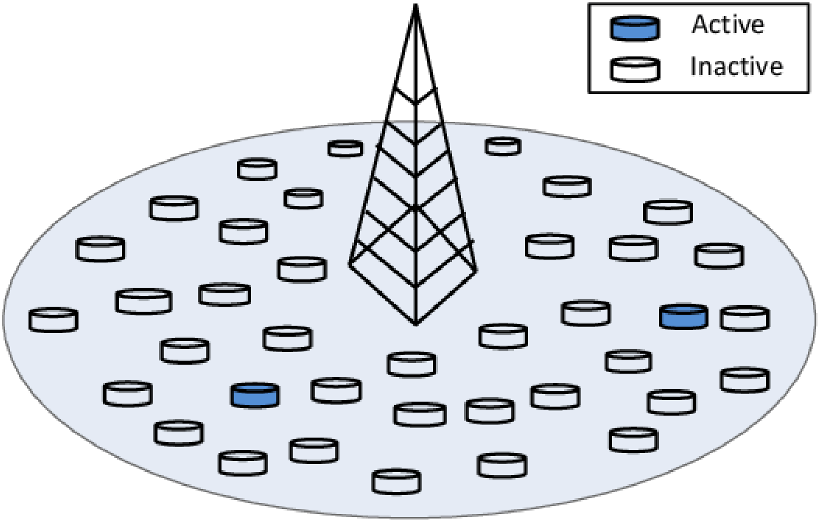
- Remote surgery.
- Autonomous driving.
- Factory automation.

Random Access in mMTC



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- Minimum chordal distance:

$$\delta_c(\mathcal{S}) = \sqrt{1 - \mu^2(\mathcal{S})}.$$

Codes for massive random access (II)

Binary Chirps: © R. Calderbank, D. Howard, J. Searle

- Fix $m \in \mathbb{N}$ and $N = 2^m$.
- Fix $\mathbf{b} \in \mathbb{F}_2^m$ and $\mathbf{S} \in \mathrm{Sym}(\mathbb{F}_2, m)$.

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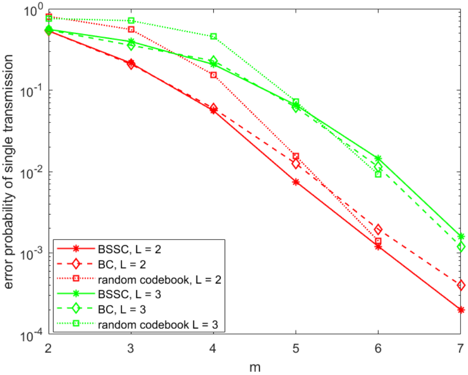

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- M users, L active users, $L \ll M$.
- Each active user u_ℓ transmits signal $\mathbf{s}_{u_\ell} \in \mathbb{C}^{N \times n_t}$, $\|\mathbf{s}_{u_\ell}\|_F = 1$.
- Receiver sees

$$\mathbf{s} = \left(\sum_{\ell=1}^{L} \mathbf{s}_{u_\ell} \mathbf{c}_\ell \right) + \mathbf{n}, \quad \mathbf{c}_\ell \in \mathbb{C}^{n_t \times n_r}, \mathbf{n} \in \mathbb{C}^{N \times n_r}.$$

- **Problem:** Determine $\{u_1, \ldots, u_L\}$ given $\mathbf{s}$.

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Definition (Well-Balanced MIMO Signature Codes).

A MIMO signature code $\mathcal{C} \subset M_{N\times n_t}(\mathbb{C})$ is called *well-balanced* if for every $X_i \in \mathcal{C}$ we have

$$X_j^\dagger X_i = c_{i,j} U_{i,j},$$

where $U_{i,j}$ is an $n_t \times n_t$ unitary matrix and $c_{i,j}$ is a scalar. The signature code will be called *ε-well-balanced* if $|c_{i,i}| = 1/n_t$ and $|c_{i,j}| \leq \varepsilon$ for $i \neq j$.

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☞ **Provable recovery guaranties!**

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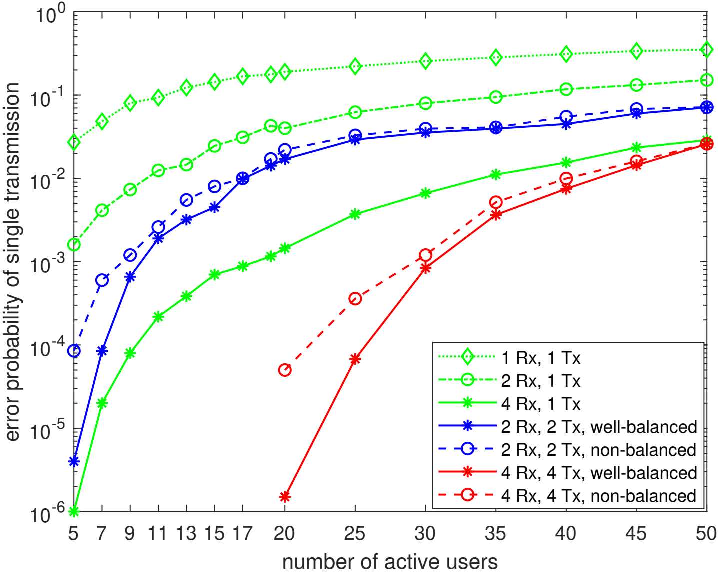

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Thank You!