Quantum Private Information Retrieval

Private Information Retrieval with Entangled Servers

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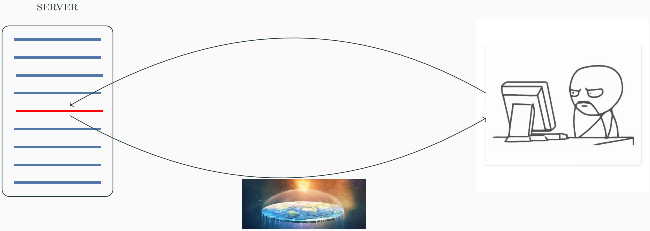
Online Privacy



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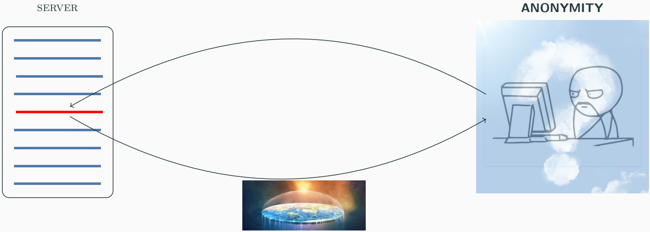

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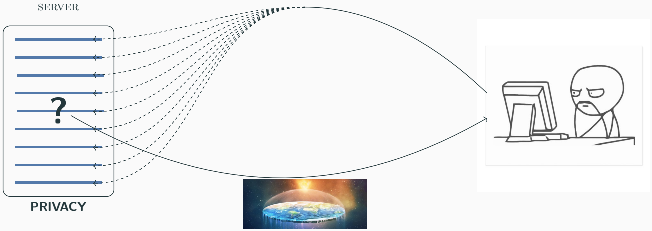

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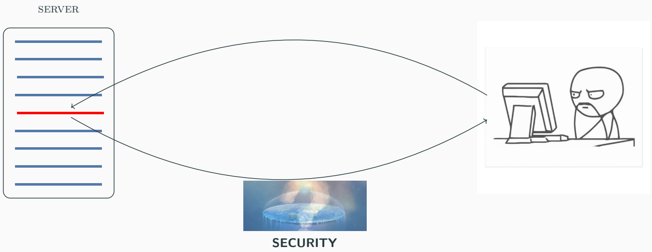

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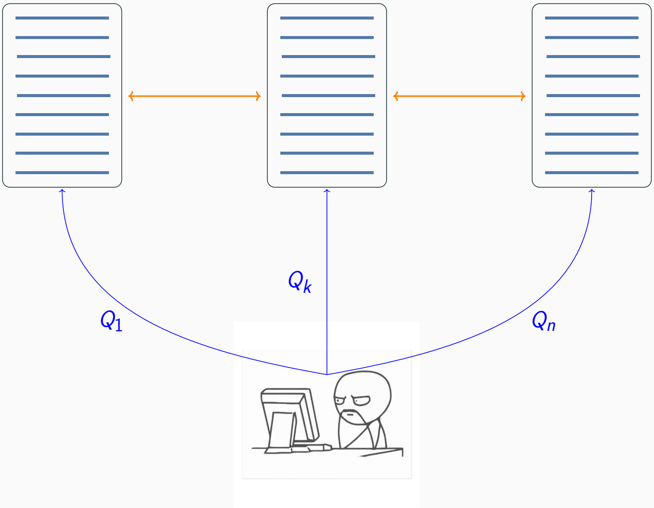
- Quest for practical solutions continues.

Coded Storage

m files $x^1, \ldots, x^m \in \mathbb{F}_q^{\beta \times k}$ are encoded and stored on n servers by a $[n, k]$ storage code $\mathcal{C}$.

$$\begin{array}{r} \\ \text{file } 1 \\ \\ \\ \\ \\ \text{file } m \\ \\ \end{array} \begin{pmatrix} x^1_{1,1} & \cdots & x^1_{1,k} \\ \vdots & \ddots & \vdots \\ x^1_{\beta,1} & \cdots & x^1_{\beta,k} \\ \vdots & \vdots & \vdots \\ x^m_{1,1} & \cdots & x^m_{1,k} \\ \vdots & \ddots & \vdots \\ x^m_{\beta,1} & \cdots & x^m_{\beta,k} \end{pmatrix} \cdot \mathbf{G}_{\mathcal{C}} = \begin{array}{c} \begin{pmatrix} y^1_{1,1} & \cdots & y^1_{1,n} \\ \vdots & \ddots & \vdots \\ y^1_{\beta,1} & \cdots & y^1_{\beta,n} \\ \vdots & \vdots & \vdots \\ y^m_{1,1} & \cdots & y^m_{1,n} \\ \vdots & \ddots & \vdots \\ y^m_{\beta,1} & \cdots & y^m_{\beta,n} \end{pmatrix} \\ \begin{array}{ccc} \mathrm{SERVER}_1 & & \mathrm{SERVER}_n \end{array} \end{array}$$

Private Information Retrieval (PIR)



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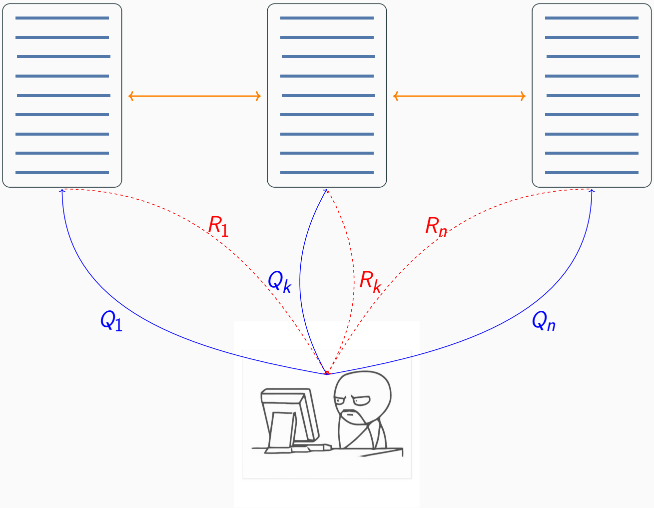

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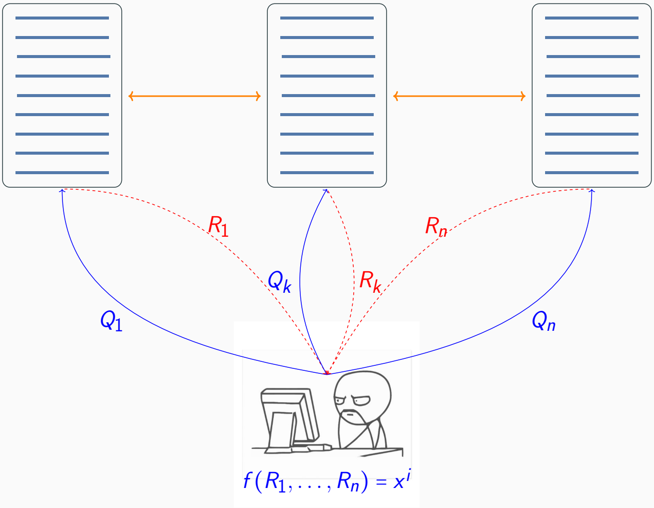

PIR with t-collusion (t-PIR)

Definition (t-PIR).

User privacy: Any set of at most t colluding nodes learns no information about the index i of the desired file, *i.e.*, the mutual information

$$I(i; Q_{\mathcal{T}}^K, R_{\mathcal{T}}^K, y_{\mathcal{T}}) = 0, \quad \forall\ \mathcal{T} \subset [n], |\mathcal{T}| \leq t\ .$$

Server privacy: The user does not learn any information about the files other than the requested one, *i.e.*,

$$I(x^j; Q^K, R^K, K) = 0, \quad \forall j \neq K\ .$$

A scheme with both user and server privacy is called *symmetric*.

PIR with t-collusion (t-PIR)

Definition (Rate and Capacity).

For a PIR scheme the **rate** is the number of information bits of the requested file retrieved per downloaded bits, *i.e.*,

$$R_{\text{PIR}} = \frac{\text{Number of bits in a file}}{\text{Number of downloaded bits}} .$$

The PIR **capacity** is the supremum of PIR rates of all possible PIR schemes, for a fixed parameter setting.

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- Replicated storage with $t = n - 1$ collusion.
- **Goal**: $[n, k]$ coded storage with $t = n - k$ collusion.

Ingredients for QPIR

- **Star Product** PIR scheme from Freij-Hollanti et al.
  - Coded storage with storage code $\mathcal{C}$.
  - A **retrieval** code $\mathcal{D}$ that determines the privacy.
  - Scheme with **rate** $(d_{\mathcal{C}\star\mathcal{D}}-1)/n$ that protects against $d_{\mathcal{D}^\perp}-1$ **collusions**.

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- **Quantum Computation**.

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- **The PVM**
  $\mathcal{B}_{\mathbb{F}_2^2} = \{\mathbf{B}_{(a,b)} = \mathbf{W}_1(a, b)|\Phi\rangle\langle\Phi|\mathbf{W}_1(a, b)^{\mathrm{t}} \mid a, b \in \mathbb{F}_2\}$.

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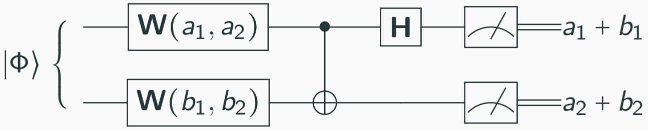
- **Two-Sum Protocol**: Alice and Bob send the sum (a_1+b_1, a_2+b_2) of their bits to Carol.



A QPIR Example

- $n = 4$ servers and $[4,2]_4$ - coded database with RS code

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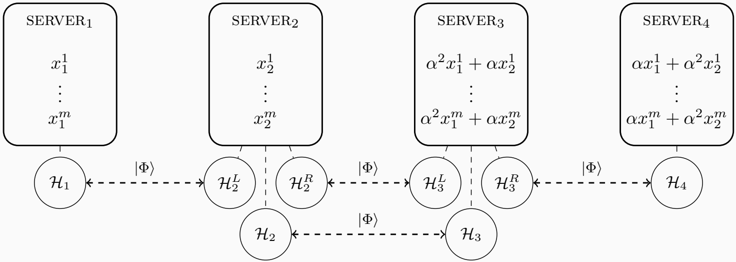
- Query index K, i.e., the requested file is $x^K = (x_1^K, x_2^K)$.

A QPIR Example: Entangled Servers



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- Query Q_s to server s.

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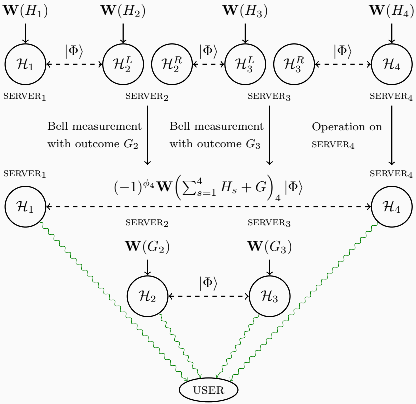

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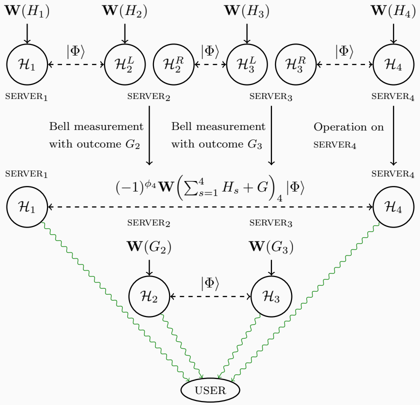

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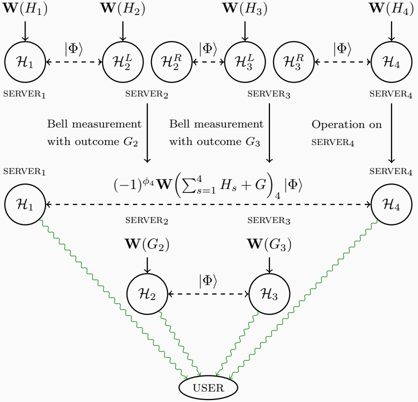

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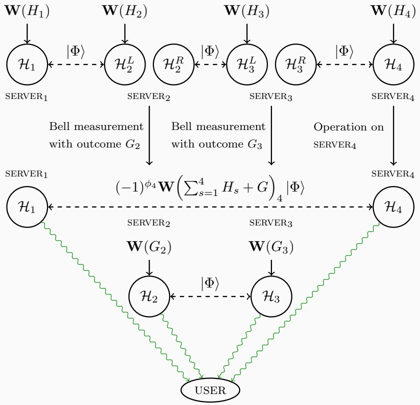

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- Each server sends its qubit to the user.

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Remark

Here we targeted servers 1&2 (systematic encoding). Since the storage is MDS-coded, one can target any two (k in general) servers.

A QPIR Example: Secrecy, Collusion, Rate

- **User secrecy**: queries $Q_1, \ldots, Q_4$ independent of the index K, two random vectors generated and encoded into queries $\Rightarrow$ at least three servers needed in order to retrieve the file requested $\Rightarrow$ **2-collusion**.

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- **Rate**: $R = \frac{2 \cdot 2}{2 \cdot 4} = \frac{1}{2}$.

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- Query index: K.

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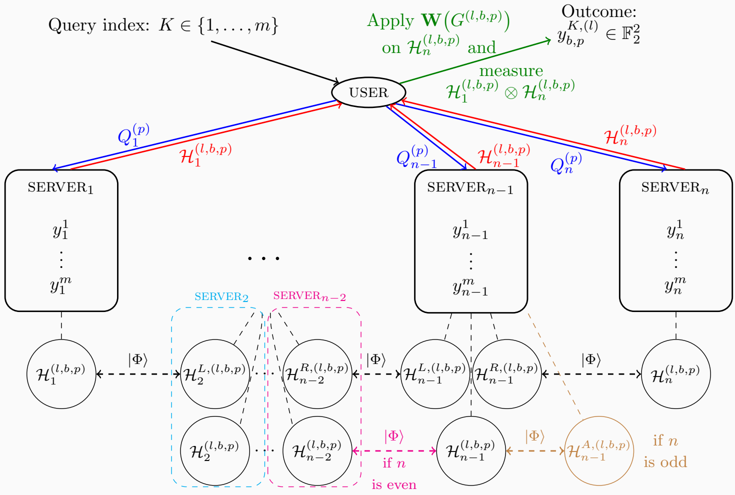

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- Server secrecy: **symmetric PIR scheme**.

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- Server secrecy: symmetric PIR scheme.
- Upload cost negligible to the file size.
- **Rate**: With $n = k + t$

$$R_{\text{PIR}} = \begin{cases} \frac{2}{n}, & \text{if } n \text{ is even,} \\ \frac{2}{n+1}, & \text{if } n \text{ is odd,} \end{cases}$$

Improvements with Locally Repairable Codes (LRC)

Definition

An $[n, k]$ code $\mathcal{C}$ is said to have (r, ρ)-locality if there exists a partition $\mathcal{P} = \{\mathcal{A}_1, \ldots, \mathcal{A}_\mu\}$ of $[n]$ into sets $\mathcal{A}_l$ with $\mathcal{A}_l \leq r + \rho - 1, \ \forall l \in [\mu]$ such that for the distance of the code restricted to the positions indexed by $\mathcal{A}_l$ it holds that $d(\mathcal{C}_{\mathcal{A}_l}) \geq \rho, \ \forall \ l \in [\mu]$.

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Optimal LRC achieve the **Singleton-like bound**

$$d \leq n - k + 1 - \left(\left\lceil \frac{k}{r} \right\rceil - 1\right)(\rho - 1).$$

Improvements with Locally Repairable Codes (LRC)

Definition

An $[n, k]$ code $\mathcal{C}$ is said to have (r, ρ)-locality if there exists a partition $\mathcal{P} = \{\mathcal{A}_1, \ldots, \mathcal{A}_\mu\}$ of $[n]$ into sets $\mathcal{A}_l$ with $\mathcal{A}_l \leq r + \rho - 1, \; \forall l \in [\mu]$ such that for the distance of the code restricted to the positions indexed by $\mathcal{A}_l$ it holds that $d(\mathcal{C}_{\mathcal{A}_l}) \geq \rho, \; \forall \; l \in [\mu]$.

Optimal LRC achieve the Singleton-like bound

$$d \leq n - k + 1 - \left(\left\lceil \frac{k}{r} \right\rceil - 1 \right) (\rho - 1).$$

The local codes $\mathcal{C}_{\mathcal{A}_l}$ of an optimal LRC $\mathcal{C}$ are $[r + \rho - 1, r]$-MDS.

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For $t = \rho - 1$, the retrieval rate of LRC-based QPIR scheme is

$$R_{\mathrm{QPIR}} = \begin{cases} \frac{2}{r+t}, & \text{if } r+t \text{ is even,} \\ \frac{2}{r+t+1}, & \text{if } r+t \text{ is odd,} \end{cases}.$$

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- Improved retrieval rate.
- Trade-off with server collusion/failure.
  - $t = \rho - 1$ colluding nodes, provided that no more than t nodes collude per local group.
  - For such collusion patterns, the scheme can resist collusion of up to $t\mu = (\rho - 1)\mu$ servers

Thank You!